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# MESON SYMMETRY* 

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(Received 25 March 1965)

In this Letter we present evidence for a broen chiral symmetry in the mass spectrum of mesonic states, and an interpretation of all well-established mesons in terms of a single family of Regge trajectories associated with the representation $\left(\underline{3}, \underline{3}^{*}\right) \oplus\left(\underline{3}^{*}, \underline{3}\right)$ of the group $\operatorname{SU}(3)_{L} \otimes \mathrm{SU}(3)_{R}$. The mesons are interpreted as bound states, primarily of baryon-antibaryon systems.

In Fig. 1 are represented the mesonic states known to date, ${ }^{1}$ on a plot of angular momentum $J$ vs (mass) ${ }^{2}=t .^{2}$ According to theoretical ideas concerning bound states, we associated a Regge trajectory with each particle and resonance. Some of the high-ranking poles have apparently been observed in the region $t<0$ through highenergy crossed-channel reactions such as elastic scattering $\left(P, \omega, P^{\prime}\right),{ }^{3}$ charge exchange $(\rho, R),{ }^{4}$ and associated production ( $K^{*}$ ). These, with $Q$ [deduced from $\operatorname{SU}(3)$ symmetry or bootstrap dynamics ${ }^{5}$ ] and $\varphi$ (a trajectory must exist associated with this particle although nothing is known about its intercept), are also indicated near $t=0$ in Fig. 1.

In general, as exchange potentials are intrinsic in crossing-symmetric relativistic problems, physical states will recur at intervals


FIG. 1. Known meson states and $B=0$ Regge trajectories with suggested $\operatorname{SU}(3)$ classification.
$\Delta J=2$ along a given trajectory. No two physical mesons are observed which would be expected to lie on the same trajectory, assuming the slopes are all less than unity (in units of $\mathrm{BeV}^{-2}$ ), as suggested both by elastic scattering analyses ${ }^{3}$ and theoretical estimates. ${ }^{6}$ However, there is now evidence ${ }^{7,8}$ that $A_{2}$ and $K^{*}(1.4 \mathrm{BeV})$ have $J^{P}=2^{+}$, which is suggestive of the possibility that they may be physical states lying on the $(R, Q)$ trajectories roughly paralleling those of the known vector mesons.

Considering $B \bar{B}$ bound states in a bootstrap picture, in which the binding potentials are principally due to meson exchanges, ${ }^{9}$ one would expect the exchange part of the effective potential (arising from baryon-number two states) to be weak compared to the direct terms. This leads to a picture in which the trajectories of the vector octet $+\operatorname{singlet}\left[\rho, K^{*}(0.890), \varphi, \omega\right]$ are almost degenerate with a set of trajectories of opposite signature. This second set would be expected to have opposite $G$ parity, when eigenstates of $G$ are considered, since at least in the $\bar{N} N$ system $G=(-1)^{s+I+l, ~ a n d ~ t h e ~ t r a-~}$ jectories may be characterized by definite $I$, $s$, and parity. ${ }^{10}$ We therefore associate the vector octet trajectories with another octet which is to appear at $J^{P}=2^{+}$. Presumably this is $A_{2}\left(I^{G}=1^{-}\right), K^{*}(1.4)$ with $I=\frac{1}{2}$, and a third yet-to-be-discovered isosinglet ( $2^{+}, 0^{+}$) meson $f^{0 \prime}$ with mass around 1.5 BeV , dynamical companion of the $\left(1^{-}, 0^{-}\right) \varphi$ meson.

The $\mathrm{SU}(3)$ singlet ${ }^{11}$ Pomeranchuk trajectory, including $f^{0}$, will have as its dynamical companion the singlet $\omega$ trajectory. Both singlet and octet trajectory pairs have $1^{-}, 2^{+}$, etc., states as physical possibilities, but not $1^{+}, 2^{-}$, etc. We will use the terminology positive-parity trajectory for the former case, and negative for the latter. The approximate dynamical degeneracy of the two sets of trajectories with opposite $G$ parity and signature will be termed exchange degeneracy.

Turning to the $0^{-}$mesons, with negativeparity trajectories, we expect (if exchange degeneracy is valid there also) the octet ( $\pi, K, \eta$ ) to be accompanied by a companion $1^{+}$octet with masses between 1.0 and 1.3 BeV . In this region lie the $A_{1}, C$, and $B$ mesons. The $B$ meson ${ }^{1}$ is known to have $I^{G}=1^{+}$, which is as required for a companion of $\pi$; the $A_{1}$ meson $^{7}$ probably has $J^{P}=1^{+}$, although $I$ and $G$ have not been determined. To form an octet we need a member with $I=\frac{1}{2}$ and $J^{P}=1^{+}$, for which $C^{12}$ is a possible candidate. Assuming these three do in fact make up an octet, we require, in addition, $I^{G}=0^{-}$for the $A_{1}$, and definitely $J^{P}$ $=1^{+}$for $B$. Note that our assignments require no $\pi \eta$ decay mode of $A_{1}$, but allow that mode for $A_{2}$.

The remaining known $0^{-}$meson, the $X^{0}$ at 960 MeV , we tentatively assign to an $\mathrm{SU}(3)$ singlet classification. ${ }^{13}$ We expect then a companion $1^{+}$state with mass around 1.4 BeV . There is one more known resonance, with roughly this mass, known ${ }^{14}$ to decay into $K \bar{K} \pi$; thus its trajectory will have negative parity, and is the best bet for this position. Since the $X^{0}$ has $I^{G}$ $=0^{+}$, this analog must have $I^{G}=0^{-}$.

We will not speculate on possible $2^{-}$recurrences around 2 BeV of the negative-parity trajectories, as little experimental evidence exists in that region.

Reasoning of an inductive nature leads then to the assignments indicated in Fig. 1. If we consider the situation without $\mathrm{SU}(3)$ or exchange splitting, the trajectories degenerate as shown in Fig. 2. At this point we may look for further symmetries which have up to now been hidden by the complexity of the observed mass spectrum and incomplete experimental data.

We have accounted for all the well-established low-lying mesonic states, with only one reso-


FIG. 2. Degenerate form of meson trajectories with unbroken $\mathrm{SU}(3)$ and exchange symmetries (solid lines), and with suggested full symmetry (dotted line).
nance yet to be verified, assuming all the suggested quantum-number assignments hold up. The resulting set of trajectories is highly suggestive of a remaining broken chiral symmetry, ${ }^{15}$ linking positive- and negative-parity states, which would yield the completely degenerate 36 -fold trajectory shown as a dotted line in Fig. 2 if all symmetry breaking could be suppressed. One possible symmetry group which could be invoked at this stage is $\operatorname{SU}(3)_{L} \otimes \mathrm{SU}(3)_{R}$, as described by Gell-Mann. ${ }^{18}$ Representations of this group which provide parity eigenstates include $\left(\underline{3}, \underline{3}^{*}\right) \oplus\left(\underline{3}^{*}, \underline{3}\right)$, which under $\operatorname{SU}(3)$ transformations only reduces to a singlet + octet with positive parity together with a similar pair of negative parity. This is exactly the pattern observed in the meson trajectories, or alternatively in the $J=1$ physical mesons, if we assume the full symmetry suggested by Fig. 2. There is no obvious pattern of mass splitting in Fig. 2, however, and without further theoretical work concerning the relation of dynamics to the suggested symmetry, it seems no mass formula will be available.

We note the following observations on the above symmetry scheme:
(1) One may compare the suggested symmetry group with other current schemes such as SU(6) and its relativistic generalizations, e.g. $\tilde{U}(12)$. In that type of symmetry there is a degeneracy between Yukawa-type couplings of $0^{-}$mesons and those of $1^{-}$mesons, as well as common mass-splitting terms in the $\mathrm{SU}(3)$ breaking mass formula. ${ }^{17}$ The latter would carry through in our symmetry if the trajectories remain straight parallel lines after chiral symmetry is broken; the former would obtain only if the Regge residues were constant along trajectories. These circumstances might be expected to hold in the limit of very large effective mass of the baryons (or baryonic states) which we have imagined bound together to give the mesons. This seems similar to the situation in which $\operatorname{SU}(6)$ is considered to be a good symmetry only in the strong-coupling, infinitemass limit of a quark model. ${ }^{18}$ Note that Fig. 2 suggests the effective central mass of that the $J=1$ mesons is 1.00 BeV , and if $\mathrm{SU}(6)$-type symmetry is good, this is just what is required to obtain the correct numerical value for the magnetic moments of the (physical) baryons. ${ }^{19}$
(2) No $0^{+}$mesons, such as $\sigma$ and $\kappa$, are predicted in the physical mass spectrum; it would appear from Fig. 2 that even the $0^{-}$mesons
might be expected to disappear in the limit of exact chiral symmetry. This limit may in fact force the degenerate trajectory to pass through $J=0$ at $t=0$, if the reasoning of Nambu and coworkers ${ }^{20}$ based on Goldberger-Treiman-type relations is relevant.
(3) No $I \geqslant \frac{3}{2}$ trajectories pass above $J=0$ at $t=0$. This means that if peripheral processes with $I \geqslant \frac{3}{2}$ exchange are observed, the magnitude of the forward peak (such as observed at $1.7 \mathrm{BeV} / c$ in $K^{-}+p \rightarrow \Sigma^{-}+\pi^{+}$) must decrease more rapidly with energy than similar reactions which can proceed through $K^{*}(0.890)$ exchange, assuming the highest Regge pole in the crossed channel dominates the reaction.
(4) The $f^{0,}$ might decay preferentially into $K \bar{K}$, if it resembles the $\varphi$ dynamically, as implied by our picture of trajectories.
(5) Relations between two-body decay widths of $A_{1}$ and $C$ may be obtained in the limit of $\mathrm{SU}(3)$ symmetry, just as $\rho \rightarrow 2 \pi$ and $K^{*} \rightarrow K+\pi$ are related. This is a consequence merely of the octet assignment for these mesons. Additional observable relations, such as between $\rho \rightarrow 2 \pi$ and $A_{2}-\rho+\pi$, will hold only if the $\operatorname{SU}(6)$-type symmetry is a good one.

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